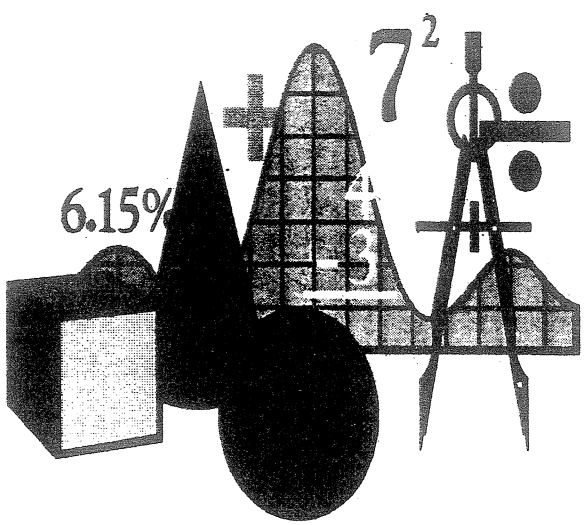
MINISTRY OF EDUCATION,

WOMEN & CULTURE

FIJI SEVENTH FORM CERTIFICATE

MATHEMATICS



CURRICULUM DEVELOPMENT UNIT
1997

MINISTRY OF EDUCATION, WOMEN & CULTURE

FIJI SEVENTH FORM EXAMINATION

MATHEMATICS PRESCRIPTION

(Revised 1997)

1.0 PREAMBLE

1.1 The Seventh Form Mathematics prescription forms a part of the continuing development of the mainstream of Mathematics. It is designed to prepare students for tertiary studies in Mathematics and also to equip those who will complete their formal education at the end of the Seventh Form. The broad-based prescription is to make provision for students who will need Mathematics for the scientific and technological world as well as for those who will need Mathematics for the commercial sector.

2.0 AIMS AND OBJECTIVES

2.1 AIMS

2.1.1 The main aims of the course are to help students to

(a) gain a better understanding of mathematical concepts and their applications

(b) acquire a much broader mathematical background that will enable them to proceed to higher education in mathematics and other subjects

(c) appreciate the role of mathematics as a tool in our every-day existence.

2.2 OBJECTIVES

2.2.1 On completing the course, students should be able to

(a) refine their ability to apply their mathematical skills and knowledge in solving problems drawn from the physical, social and commercial environments

(b) make better use of mathematical instruments such as mathematical tables, calculators and computers in problem solving

(c) show greater competence in curve sketching and showing relationships of variables graphically

(d) display a more logical approach to problem solving

(e) classify information as either being relevant or irrelevant to the solving of a given problem

(f) actively involve themselves in mathematical processes such as problem solving, logical reasoning, making connections, communicating results and findings and using tools.

3.0 COURSE CONTENT : OUTLINE

The prescription consists of the following ten topics. Topics 1 to 9 are compulsory and there is a choice in Topic 10.

Topic One	Algebra	
Topic Two	Limits and Continuity	
Topic Three	Functions	
Topic Four	Trigonometry	
Topic Five	Probability & Statistics	
Topic Six	Calculus I	
Topic Seven	Series and Convergence	
Topic Eight	Complex Numbers	
Topic Nine	Calculus II	
Topic Ten	Either	Vectors & Geometry
	OR	Computing.

4.0 COURSE CONTENT : DETAIL

4.1 TOPIC ONE - ALGEBRA

4.1.1. Inductive definition, Proof by Induction

4.1.2 Binomial Theorem.

EXPLANATORY NOTES

[i] Students should be introduced to a formal definition of mathematical induction.
The inductive method will also be dealt with in other parts of the syllabus.

[ii] The Binomial Theorem is to be approached through a study of the patterns evident in Pascal's triangle which relates to the expansion of $(x + y)^n$

Students need to be familiar with the work on factorials and combinations and the evaluation of the coefficients in the expansion of $(x + y)^n$
Proof will only be required for positive integrals.

4.2 TOPIC TWO - LIMITS AND CONTINUITY

4.2.1 Limits

4.2.2 Continuity

EXPLANATORY NOTES

[i] The ideas of limits and continuity should be treated intuitively by the use of diagrams and numerical examples.

[ii] Continuity need not go beyond the 'unbroken line' concept.

The following should be considered :

(a) discontinuities arising from the function being undefined at a point, e.g.

$$\frac{1}{x - 3} \qquad \text{at } x = 3$$

(b) discontinuities in the derivative, e.g.

$$|x| \qquad \text{at } x = 0$$

4.3 TOPIC THREE - FUNCTIONS

4.3.1 Behaviour of polynomial and rational functions.

4.3.2 Algebra of functions; sum and difference of functions, composite functions.

4.3.3 Partial fractions.

EXPLANATORY NOTES

[i] Topic 3 extends the study of polynomials and rational functions covered in the Fiji School Leaving Certificate course. Results such as the factor and remainder theorems for polynomials continue to have applications. The discussion should include zeros, asymptotes, local maximum and minimum values, ranges of values, behaviour 'at infinity' and particularly the graphical representation of these aspects.

[ii] The term 'simple rational function' in the prescription means one in which the sum of the degrees of the numerator and denominator is four or less. All real factors of cubic and quadratic polynomials should be given explicitly; this includes numerators and denominators of rational functions.

[iii] The work on algebra of functions should be mainly on the sum and difference of functions (f+g, f-g) and composite functions (fog). Students should also be required to derive the domain of f+g, f-g and fog.

[iv] Simple partial fractions should be of the type where, in the denominator, there are

- distinct linear factors
- repeated linear factors
- distinct quadratic factors
- repeated quadratic factors

4.4 TOPIC FOUR - TRIGONOMETRY

4.4.1 Periodic functions and graphs.

4.4.2 Identities and proofs.

4.4.3 Sums and products of sines and cosines.

4.4.4 Half-angle formula.

4.4.5 Inverse trigonometric functions.

4.4.6 Trigonometric equations.

EXPLANATORY NOTES

[i] The function $f(x) = R \begin{matrix}\text{Sin} \\ \text{Cos}\end{matrix} (x \pm \theta)$ and the applications should include minimum and maximum values, periodicity and transformations.

[ii] A consideration of the six trigonometric functions can lead to a host of identities interconnecting them. Students should see that proving identities involves expressing trigonometric functions in other equivalent forms and applying algebraic methodologies to simplify.

[iii] Work on the Addition Laws should lead on to the Double-Angle Formula, the Half-Angle Formula and the Sum and Products of trigonometrical functions.

[iv] The inverse trigonometric functions are of the form $y = \arcsin x$, $y = \arccos x$, $y = \arctan x$ or $y = \sin^{-1} x$, $y = \cos^{-1} x$ and $y = \tan^{-1} x$. The study of inverse trigonometric functions should include their graphs, domain and range.

[v] Students should be able to reduce trigonometric equations to one of the simplest equations like $\sin\theta = k$, $\cos\theta = k$ or $\tan\theta = k$.

4.5 TOPIC FIVE PROBABILITY AND STATISTICS

4.5.1 Probability

Sample space and events, probability of an event, complementary and mutually exclusive events, independent events and the probabilities associated with these events.

4.5.2 Population and Samples

Mean, Median, Standard deviation and range as examples of population parameters : Samples, random samples, frequency distributions and sample statistics. Sampling, reasons for sampling, bias.

Methods of sampling; pilot survey, simple random, systematic, cluster, stratified and opinion polls.

4.5.3 **The Central Limit Theorem**

Distribution of the Sample Mean.
Expectation and Variance of the Sample Mean.

4.5.4 **Estimation**

Finding confidence intervals for the mean, μ and sample sizes required for estimates of μ.

4.5.5 **Distributions**

- The Binomial distribution taken as an example of a discrete distribution, mean and standard deviation of binomial distributions.
- The Normal (Gaussian) distribution as an example of a continuous distribution; Z-score.
- Inverse Normal problems

4.5.6. **Introduction to Hypothesis Testing**

- Significance levels
- Null and Alternative Hypothesis
- Type I and Type II Errors
- One tailed and two tailed tests
- Construction of Hypothesis Tests
- Hypothesis tests for means only (Hypothesis tests for proportions and the difference between 2 means is to be excluded.)

EXPLANATORY NOTES

(i) Students should have already done work on population parameters, sample statistics and sampling at Form 6 level. The emphasis at the Form 7 level should be on sections other than 4.5.2.

(ii) Binomial Distribution : For $n \geq 4$, binomial probability should be obtained from tables.

(iii) Work on the T distribution is to be excluded.

(iv) The emphasis on the teaching of this topic should focus not only on acquainting students with the theoretical concepts but also to provide experiences to them so that they could conduct basic statistical experiments or research and be able to analyse and interpret the data so obtained. This could be achieved through class experiments, research projects or with reference to published results from surveys or opinion polls. Teachers should enable their students to see the impact that modern technological tools such as calculators, and where possible computers, have in processing massive amounts of information and data.

(v) The topic Probability and Statistics is recommended to be taught in Term I as outlined because the ideas and concepts learnt in this topic would assist students in doing research projects in other subject areas.

4.6 TOPIC SIX - CALCULUS I

DIFFERENTIATION

4.6.1 The derivative defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

4.6.2 Differentiability at a given point.

4.6.3 Differentiation of sums, products, quotients, composite functions, circular functions (excluding inverse trigonometric functions), logarithmic and exponential functions, implicit relations.

4.6.4 Derivatives of second and higher orders.

INTEGRATION

4.6.5 The definite integral as the limit of a sum.

4.6.6 Indefinite integration as the inverse of differentiation.

4.6.7 Integration by parts.

4.6.8 Integration by simple substitutions (excluding trigonometric substitutions).

EXPLANATORY NOTES

[i] The topic CALCULUS I should consolidate and extend the work done at Sixth Form Level. It concentrates on the concept of functions and the emphasis here should be the building of the skills of differentiating and integrating a variety of function types.

[ii] Applications of differentiation and integration should be discussed in relation to maxima and minima, areas, related rates and points of inflexion.

[iii] The connection between the definite and indefinite integrals should be discussed. This discussion should include the Fundamental Theorem of Calculus.

[iv] The applications of the definite integral should include areas and mean values of a function.

4.7 TOPIC SEVEN SERIES AND CONVERGENCE

4.7.1 Sigma Notation.

4.7.2 Series and Sequences.

4.7.3 Convergence.

EXPLANATORY NOTES

[i] Expressing a sum in sigma notation should involve both an infinite sum and a finite sum. Evaluation of a finite sum should include, wherever possible :

- summation of individual terms for the purpose of applying an appropriate formula
- re-adjustment of limits to start from unity
- cancellation of terms.

[ii] The study of sequences as real-valued functions on the set of natural numbers should include the general term of a sequence, infinite sequence of partial sums and general term of a series.

[iii] The study of convergence should be illustrated by the geometric, binomial, exponential and logarithmic series and such series as are immediately comparable with them. The non-convergence of (1/n) and arithmetic series should be included. [All formal tests for convergence and divergence are excluded.]

[iv] Series such as the binomial, logarithmic and exponential series should be studied.

4.8 TOPIC EIGHT COMPLEX NUMBERS

4.8.1 Complex numbers including conjugates.

4.8.2 Argand diagrams, modulus and arguments.

4.8.3 Polar form of complex numbers.

4.8.4 De Moivre's Theorem for rational exponents.

EXPLANATORY NOTES

[i] The work in algebraic structures in relation to complex numbers need not be covered.

[ii] The complex exponential function and mappings of complex numbers will not be examined.

4.9 TOPIC NINE CALCULUS II

4.9.1 Differentiation of inverse sine, cosine and tangent functions.

4.9.2 Integration using trigonometric substitutions of the type $x = a \sin\theta$ etc.

4.9.3 Application of differentiation and integration to include work on volumes of revolution (both along x- and y - axis) and as applied to kinematics.

4.9.4 Differential Equations : an introduction to 1st order differential equations only with reference to solving real life problems as applied to exponential growth and decay, kinematics, commerce (compound interest, inflation, depreciation) and science (population growth)

EXPLANATORY NOTES

Calculus II is an extension of the work covered in Calculus I. The concepts and skills for differentiating and integrating are developed further to look at a wider range of functions. This section puts emphasis on the more difficult aspects of Calculus including harder applications.

4.10 **TOPIC TEN**

4.10.1 Either : **OPTION A VECTORS & GEOMETRY**

(i) Vectors in 2 and 3 dimensions

(ii) Scalar products and orthogonality

(iii) Vector representation of a straight line

EXPLANATORY NOTES

Treatment of Vectors should include multiplication by a scalar, addition and subtraction, the ratio theorem, unit vectors, components, scalar product of two vectors, its geometrical significance and algebraic properties, position vectors, the equation of a straight line ($x = a + bt$). The equation of a plane and linear dependence of vectors is excluded.

4.10.2 OR **OPTION B COMPUTING**

Components of a computer system, developments in computer technology, microcomputer applications : algorithms and flowcharting, programming in BASIC.

EXPLANATORY NOTES

- The study of flowcharts should involve flow diagrams, branch programmes, loop programmes and a practical application of a branch programme.

- Students should be introduced to the BASIC language and to the use of the following BASIC commands :

  LET, INPUT, READ/DATA, PRINT, IF/THEN, FOR/NEXT, GOSUB/RETURN, GO TO, END, to write programmes.

  Students should be fully aware of the flowchart symbols for :

  - start and end
  - read and print
  - process
  - condition
  - for/next
  - a connector

  Students should know the meanings and effects of the following BASIC commands :

  - delete
  - edit
  - files
  - list
  - load
  - save
  - run

  The use of the following functions should also be considered :

  SIN(X), COS(X), TAN(X), EXP(X), LOG(X) and SQR(X) in simple programmes.

5.0 TIME ALLOCATION

It is expected that schools will allocate at least 5 hours per week to this course. This is to cover both lectures and tutorials. Tutorials should be at least one period per week.

The recommended minimum time schedule for each topic is given below.

TOPIC	NO. OF WEEKS
1. Algebra	2
2. Limits and Continuity	1½
3. Functions	2½
4. Trigonometry	3½
5. Probability & Statistics	4
6. Calculus I	4
7. Series and Convergence	3
8. Complex Numbers	3
9. Calculus II	2½
10. Optional (Vectors & Geometry/Computing)	2
	28

6.0 EVALUATION

Evaluation will be in the form of a three-hour written examination. The examination will cover all the topics 1 to 10.

The examination paper will be divided into three sections A, B and C.

Section A will be based on Topics 1 - 8 and will carry 25% of the total possible marks. There will be 10 questions each worth 2.5 marks. All questions will be compulsory.

Section B will also be based on Topics 1 - 8 and will carry 60% of the total possible marks. There will be 5 questions worth 12 marks each. All questions will be compulsory.

Section C will be based on Topics 9 and 10 only and will carry 15% of the total possible marks. Students will be required to do Question 1 on Calculus II which is compulsory, and either Question 2 on Vectors & Geometry or Question 3 on Computing. Each question is worth 7½ marks.

7.0 RECOMMENDED TEXTS

1. Searly, J. R., Agnew, A. W. (1980) Advanced Mathematics. Longman Paul.

2. Barton, Johnson, Laird. Delta Mathematics. (1989) Longman Paul.

3. Barton, D. (1986) Sigma Mathematics. Longman Paul.

4. Thong, H.S., Chiang, T.Y., Meng, K.K. (1984) College Mathematics Vol. 1. Pan Pacific Productions.

5. Chow, Y.M. et.al. (1981) College Mathematics Vol. 2. Pan Pacific Productions.

Additional Reference Books

1. Stair, R.M. and Janako, R.E. (1984). Essentials of BASIC Programming. Richard D. Irwin (ISBN : 0-256-02993-8)

2. Kreitzberg, C.B. Introduction to BASIC. Books for Professional, Inc. (ISBN : 0-15-600034-2)

3. Newmark, J. (1983). Statistics and Probability in Modern Life. New York : Saunders College Publishing.

4. Walpole, R.E. (1982). Introduction to Statistics. New York : MacMillan Publishing Company.

5. Sidebotham, T. (1992). Mathematics Revision Form 7 with Calculus. Auckland : ESA Publications.

6. Howison, P. (1992). Mathematics Revision Form 7 with Statistics. Auckland : ESA Publications.